Spherical aberration compensation plates

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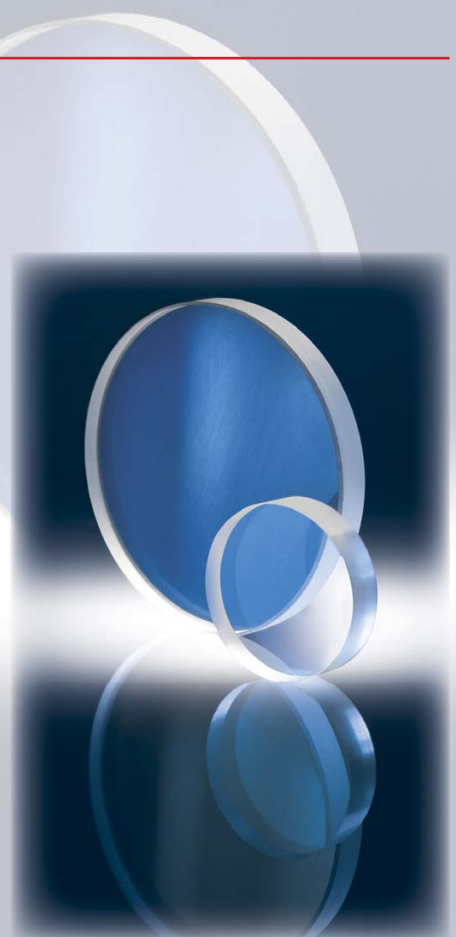**Anna Lansing, Edmund Optics GmbH, Karlsruhe, Germany**

Imaging optics strive to obtain perfect or near diffraction limited performance. The main detriment to imaging systems is spherical aberration, and it costs money and effort to eliminate or minimize. This article will outline a new optical component for controlling this aberration.

Typical optical components are standardized to focal length and diameters but not so much to their level of aberration. Within the last decade aspheric lenses have become a commodity, basically eliminating spherical aberration for a single wavelength and specific imaging parameters. The introduction of spherical aberration plates provides another tool for constructing systems that can adequately control and manipulate light.

This article will highlight typical scenarios involving spherical aberration and demonstrate how to control it. We'll discuss the magnitude of the aberration via equations and general rules of thumb. This will give the user a good understanding of how to handle spherical aberration when designing with standard components. Additionally, the article will explore comparisons between aspheres, spherical lenses and spherical aberration plates in typical user applications.

1 Theory of spherical aberration

To obtain a mathematically perfect image, a mirror requires a parabolic surface while a lens requires an elliptical surface (Lüneburg lens) [1]. For ease of manufacturing and testing, however, most optical elements have spherical surfaces. This introduces blur or a halo in the image due to spherical aberration, which results from different focal lengths at different pupil zones (**figure 1**) such as the paraxial zone close to the optical axis or the marginal zone at the edge of the lens. A typical spherical lens produces "undercorrected" spherical aberration, defined as the marginal rays focusing closer to the lens than the paraxial rays. For a collimated beam and a spherical lens, equation 1 describes the deviation W from an ideal spherical wavefront [2].

$$W = W_{040} \cdot \rho^4 \qquad \text{(Eq.1)}$$

where W_{040} is the wavefront aberration coefficient for spherical aberration (units are waves) and ρ is the normalized pupil radius. For a converging or diverging beam, the spherical term is added to a quadratic term (focus) to establish a wavefront deviation as shown in equation 2.

$$W = W_{020} \cdot \rho^2 \qquad \text{(Eq.2)}$$

where W_{020} is the wavefront aberration coefficient for focus (units are waves). Ray errors (deviation from a perfect point like image) at the image plane are a derivative of the wavefront deviation, W, with respect to the aperture, ρ. Assuming paraxial focus and neglecting diffraction effects, the total spot diameter, d, is simply the ray error of light from the edge of the focusing lens:

$$d = 16 \cdot (F/\#) \cdot W_{040} \cdot \lambda \qquad \text{(Eq.3)}$$

with wavelength λ and the lens aperture (F/#) = focal length / lens diameter. This is not the RMS (root mean square) spot diameter that is commonly used to characterize imaging performance. There are no simple equations to characterize the
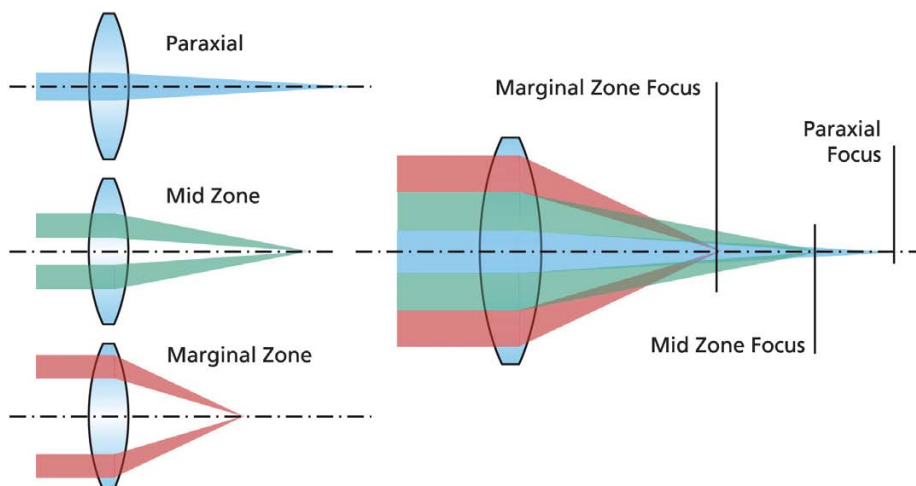

Figure 1: Simple diagram explaining spherical aberration

RMS spot size and it is usually left to ray tracing software to determine this value. d can be minimized by finding a medial focus between marginal (edge rays) and paraxial (center rays) focus, located close to "mid zone focus" in figure 1.

A lens (**figure 2**) has two radii of curvature (front and back). For any given focal length there is an infinite number of combinations, affecting aberration. A lens could be bent as a meniscus, an equiconvex or a plano convex lens. The shape factor K is
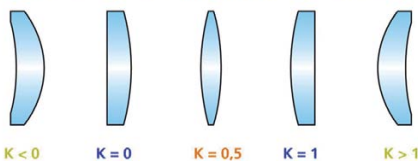

Figure 2: Singlets can be "bent" from negative to positive K

defined by

$$K = R_2 / (R_2 - R_1) \qquad \text{(Eq.4)}$$

With the index of refraction n and the lens diameter D, the amount of spherical aberration in a singlet with an infinite conjugate (object at infinity) can be computed by equation 5 [4] to

$$W_{040} = \frac{D}{128 \cdot \lambda} \cdot \frac{1}{(F/\#)^3} \cdot \frac{1}{(n-1)^2} \cdot \left[n^2 - K \cdot (2n+1) + K^2 \cdot \frac{n+2}{n} \right] \quad \text{(Eq.5)}$$

Spherical aberration in a lens with a finite conjugate is described in [3].

A positive lens (K>0) will introduce undercorrected spherical aberration and W_{040} will always be greater than zero. There are many things one can do to minimize spherical aberration and consequently shrink image size. To understand the various trades, a plot of spherical aberration versus bending and index of refraction is shown in **figure 3**. Notice how there is always a minimum to the curve. Also notice that the higher the index of refraction the lower the undercorrected spherical aberration.

There is a unique shape factor to minimize spherical aberration for each index of refraction. The first derivative of equation 5 with respect to shape factor, K, when set equal to zero will solve for the optimum shape factor (equation 6). The minimized spherical aberration is given by equation 7 and it is found by substituting the optimum shape back into equation 5.

$$K_{Optimum} = \frac{n \cdot (2n+1)}{2 \cdot (n+2)} \qquad \text{(Eq.6)}$$

and

$$W_{040 \, minimized} = \frac{D}{512 \cdot \lambda} \cdot \frac{1}{(F/\#)^3} \cdot \frac{4n^2 - n}{(n-1)^2 \cdot (n+2)} \quad \text{(Eq.7)}$$
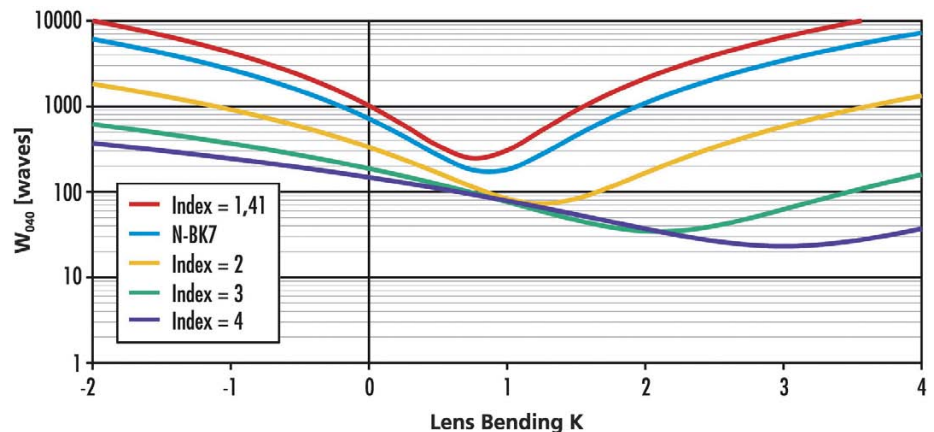

Figure 3: Singlet spherical aberration (Log) for an infinite object versus lens bending and index of refraction at 587 nm

Lord Rayleigh's criteria states that a wavefront should have a deviation < λ/4 to obtain a near diffraction limited image. Figure 3 shows that a singlet can have several orders of magnitude of spherical aberration and it shows that a singlet with the proper bending is not sufficient for good imaging. Multiple spherical lenses can be used to further reduce aberration. This modestly adds to cost but can greatly grow the size of the optical system. An asphere can be used to completely eliminate spherical aberration in the same size package as a spherical singlet. An asphere is harder to manufacture and test and it will therefore cost more than several spherical lenses.

The discussion of spherical aberration is pertinent to zero field-of-view systems such as a centered point source at infinity or laser systems. Imaging systems typically image over a field of view defined by the field stop, usually the detector. As a field of view is added, more aberrations, such as coma and astigmatism become important. These aberrations are beyond the scope of this article.

2 Spherical aberration plates

Spherical aberration plates work well in systems with little or no field of view. They are recommended for insertion in a pure collimated beam. If they are to be used in an imaging system with a field of view, it is recommended to put them near an aperture stop or a pupil. The stop is the one aperture in the optical system that limits the diameter of the beam. A pupil is an image of the aperture stop. Placing a spherical aberration plate away from a pupil will create induced coma and astigmatism and it is not recommended.

Spherical aberration plates have a surface sag departure from a flat plane that is characterized by pure ρ^4, where ρ is the semi-aperture. Spherical aberration is sign dependent and can therefore be convex or concave. A concave plate will create negative or overcorrected spherical aberration, while a convex plate will create positive or undercorrected spherical aberration.

	Convex-plano lens	Asphere	Spherical aberration plate -25 λ + convex-plano lens
Relative cost	1	8	8
Spherical aberration	21.5 λ	0 λ	-3.5 λ
Spot diameter (paraxial focus)	608 μm	1 μm	98 μm
Spot diameter (best focus)	400 μm	1 μm	36 μm
Optical layout			

Table 1: Spherical aberration content in a simple imaging example (F/3, 25 mm diameter)

Optical Components

3 Application examples

3.1 Standard imaging components

We will now contrast the use of spherical aberration plates to spherical and aspheric lenses. Since the spherical aberration plate has no optical power and cannot focus light, it has to be included with a spherical lens.

The discussion will be constrained to all glass elements: Using common fabrication methods will yield better cost and performance comparisons. Let us assume F/3 elements made from N-BK7 from Schott. Refer to **table 1** for relative cost and spherical aberration content. The best solution in terms of optical performance is a glass asphere. The lowest cost solution is a spherical element, but it has very poor performance. An intermediate solution would be a spherical lens paired with a spherical aberration plate. It has the same cost as the asphere but much improved performance over the spherical lens.

While this case doesn't compel one to use spherical aberration plates versus a standard asphere, it highlights the benefits of a spherical aberration plate added to an existing system plagued by this aberration.

3.2 A window in a converging beam

Placing a window in a collimated wavefront will not induce any additional spherical aberration and is a preferable option. However, sometimes it is necessary to place a window after a focusing lens. Typical scenario could be a protective window over a detector or bandpass filter. Placing a plane parallel plate in a focusing or converging beam will induce overcorrected spherical aberration.

The amount of spherical aberration induced in this case can be computed as follows [2,4], independent of where the window is placed in the converging beam:
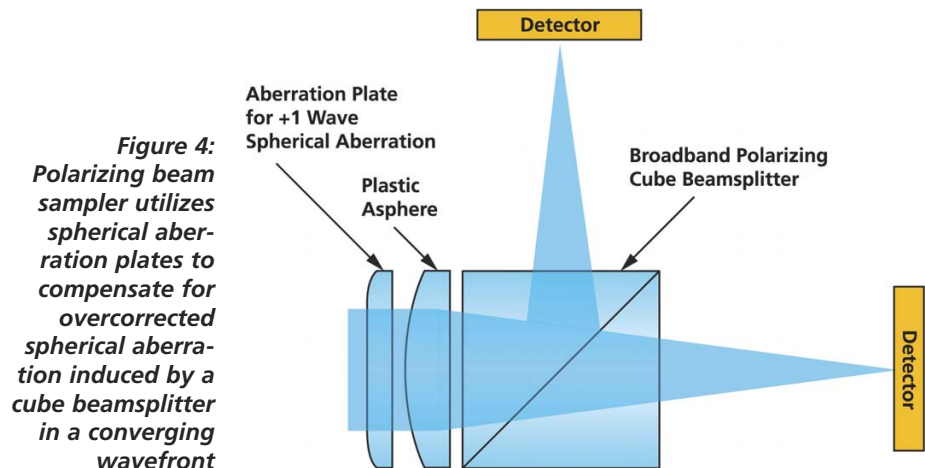

Figure 4: Polarizing beam sampler utilizes spherical aberration plates to compensate for overcorrected spherical aberration induced by a cube beamsplitter in a converging wavefront

$$W_{040} = -\frac{t}{128 \cdot \lambda} \cdot \frac{1}{(F/\#)^4} \cdot \frac{(n^2-1)}{n^3} \qquad \text{(Eq.8)}$$

3.3 Beamsplitter

When inserting a beamsplitter after a focusing lens, the user either has to tolerate the added degradation or invest into a custom asphere to compensate for the beamsplitter's induced spherical aberration. Spherical aberration plates offer a new alternative.

Let us consider a plastic aspheric lens operating at F/3.3 to focus green light onto a detector. The amount of light provides feedback for some fabrication process. The manufacturing engineer realizes that monitoring the different polarizing states will provide increased fidelity to processing. This particular plastic lens has been used in this production process for many years and management is hesitant to change the lens or use another one. The manufacturing engineer places a 25 mm broadband polarizing cube beamsplitter after the lens and adds another detector. Unfortunately the spot sizes have grown large and they overfill the detectors. The beam splitter has induced overcorrected spherical aberra-

tion. Using equation 7, the manufacturing engineer calculates the exact amount of overcorrected spherical aberration to be -1.07 waves. He places a +1 wave spherical aberration plate in front of the plastic asphere (**figure 4**). Evaluation in **table 2** shows that in this case the spherical aberration plate provides the best cost versus performance solution.

3.4 Correcting for non-standard conjugates

Most off-the-shelf lenses are designed to work with specific conjugates. Given an object distance relative to the lens, there is one unique image distance for paraxial rays. These two distances are said to be conjugate to one another. A convex plano lens (K=1) is optimum for a collimated input, where the point object is located at infinity and the image is one focal length away from the lens. A double convex lens (K=0.5) is optimum for one-to-one imaging, where the object-to-lens distance and the lens-to-image distance are equal to double the focal length. Any other set-up cannot achieve a minimum spherical aberration with standard lenses.

As an example let's de-magnify a pinhole by a factor of 20x using a single F/1.7 lens with a pinhole-to-lens distance 20 times longer than the lens-to-image distance. Ray tracing with a standard convex plano lens shows there will be +290 waves of spherical aberration. Optimizing the shape factor via software shows +135 waves of aberration at optimum bending of K=0.86. This shape factor is not a standard lens and it would have to be custom made. A standard convex plano asphere will have +6.5 waves of spherical aberration for this application. If this is too much spherical aberration, a costly custom asphere could be designed and fabricated.

	Plastic asphere + beam splitter	Custom asphere + beam splitter	Plastic asphere + beam splitter and +1 λ spherical aberration plate
Spherical aberration	-1.07 λ	0 λ	-0.07 λ
Spot diameter (paraxial focus)	28 μm	0 μm	2 μm
Spot diameter (best focus)	11 μm	0 μm	0.3 μm
Relative cost	1	9	1.5

Table 2: Optical performance for various solutions of a cube beam splitter placed after a focusing lens
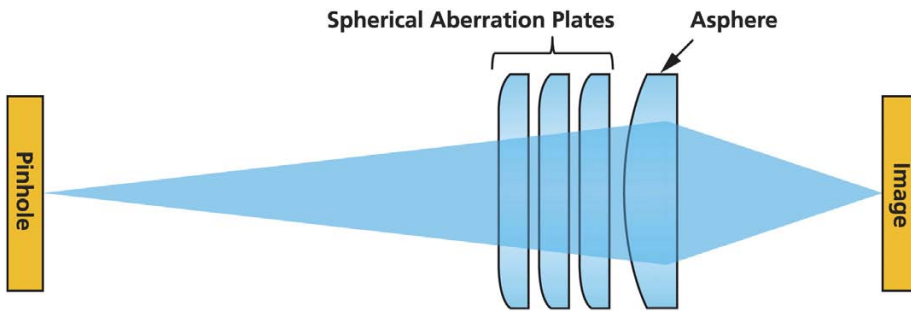
Figure 5: Spherical aberration plates (here: -5, -1 and -0.5 wavelengths, in total -6.5 wavelengths, cf. text and table 3) used to correct for non-standard imaging conjugates

Instead, one can use a standard asphere optimized for an infinite conjugate with spherical aberration plates to correct the aberrations caused by a shift in conjugates (**figure 5**). **Table 3** outlines the performance of the various optical configurations.

4 Conclusion

Spherical aberration is the zonal variation of focus and it costs money and effort to control in an optical system. It is typically minimized through lens bending or using multiple spherical components or eliminated by aspheres for an added cost. If unacceptable spherical aberration in an optical system cannot be eliminated by existing standard components, this either requires a costly custom design or new solutions involving spherical aberration plates. This provides an additional tool for the optical system designer to control aberration by off-the-shelf components.

References:

[1] R.K. Lüneburg, *Mathematical Theory of Optics*, Cambridge University Press, 1964
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[3] W.T. Welford, *Aberrations of the Symmetrical Optical System*, Academic Press, 1974
[4] W.J. Smith, *Modern Optical Engineering*, 3rd edition, McGraw Hill, 2000

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	Standard convex-plano lens	Custom bent spherical lens	Standard asphere	Custom asphere	Spherical aberration plate -6.5 λ waves* and asphere
Relative cost	1	12	8	120	30
Spherical aberration	290 λ	135 λ	6.57 λ	0 λ	0.07 λ
Spot diameter (paraxial focus)	4250 μm	1900 μm	110 μm	0 μm	1.1 μm
Spot diameter (best focus)	2200 μm	1400 μm	53 μm	<1 μm	<1 μm

Table 3: Spherical aberration content at non-standard conjugates (F/1.67, 25 mm diameter)